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MATHIFICATION

GENERICSCIENCE BADIOU, MATHIFICATION, SUBJECT

I've been returning frequently in recent weeks to that momentous section from *Being and Event* where Alain Badiou marshals all his poetic and persuasive powers. I refer to the important meditations Twenty-Six and Twenty-Seven and the "impasse of ontology" described therein, the crux of the book if not the crux of Badiou's project overall, with page 278 in the English edition containing perhaps the single most important paragraph in all of Badiou.

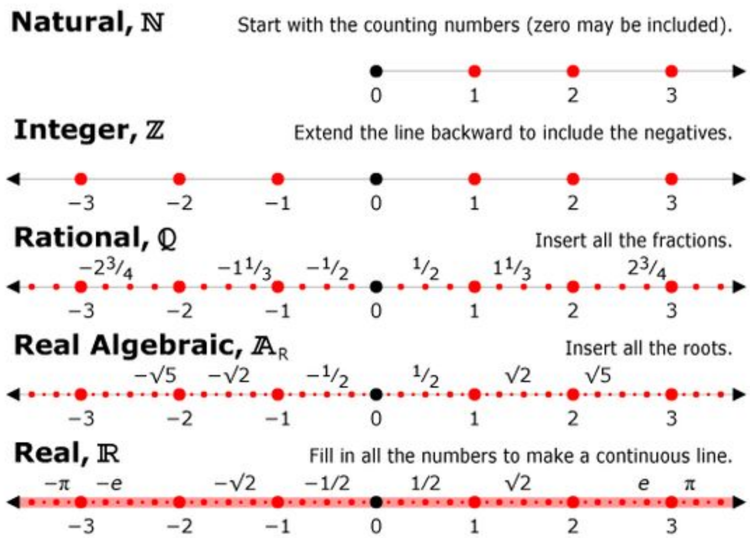
Badiou testifies in this section that the impasse of ontology was triggered world-historically by what he calls the "Cantor-Gödel-Cohen-Easton symptom," referring to the four mathematicians who together, in Badiou's assessment, have revealed a condition within mathematics, and hence also within ontology, that forces a choice (280). Cantor primarily and Cohen secondarily are the two most important figures for Badiou, particularly in *Being and Event*. Gödel figures a bit as well and Easton less so. Nevertheless Badiou combines these four figures into a single event within the history of mathematics. Badiou defines the event as an "errancy" or "excess of the state" over the situation (282). Such errancy mandates a subjective choice.

By what path does Badiou arrive at the impasse? It all begins with a query. "[I]s being intrinsically quantifiable? … Is there thus an essential numerosity of being?" (265). The query is innocent enough. Is it always possible to compare two things quantitatively? Is it always possible to say that there is something that is larger than something else? Is there a concept of "larger than" from which to construct quantity or numerosity, and, if so, is there a concept of "larger than" in thought overall? The path to the impasse begins just like that, because (as will be explained in a moment) the simple numerosity of being, the simple notion that everything is intrinsically quantifiable and therefore relatable via the operation of "larger than"–this simple reality collapsed under the weight of Cantor-Gödel-Cohen-Easton.

The impasse can be defined both in socio-political terms as well as in mathematical terms. First, in socio-political, "the state of a situation is quantitatively larger than the situation itself" (273); now again mathematically, the power set is quantitatively larger than the original set. Badiou's word is "quantitatively," but do not be mislead, the nub is that the state is immeasurably larger than the situation itself, and thus *qualitatively* larger. Likewise the fact that a power set will always be larger than its original set is a way of saying that the numerosity of the power set *qualitatively* or indeed *immeasurably* exceeds the numerosity of the original set.

This observation is perhaps insignificant or even illegible when considering finite sets, but, as Cantor showed, it becomes terribly important when considering transfinite sets.

But such claims are already too abstract. So what are some actual sets that might allow us to explore these claims, and even to approach Badiou's impasse directly? Mathematicians have a particular interest in certain kinds of sets, certain types of numbers that are important within number theory. For instance, one might consider the set of simple counting numbers like 1, 2, or 3, collectively called the *natural* numbers. Or one might wish to talk about the *integers*: 1, 2, 3, and so on, but also including zero and negative numbers like -1, -2, and -3. Or one might discuss all the fractional values like 1/2 or 15/16, these being the numbers expressible via a ratio of integers and thus gaining the title of *rational* numbers. Or one might wish to discuss an even more capacious category of numbers called the *real* numbers, those being the sum total of all of the above–the natural integers and the fractional rationals–plus everything else on the continuous number line, all the so-called irrational numbers like π that can not be written as a ratio of integers. The real numbers thus include integers like -2, fractions like 3/4, but also irrationals like π. And it turns out there are quite a lot of irrational numbers, an innumerable number of them in fact, even though only a few of them are commonly used in calculation.



Such examples are not chosen at random, particularly the natural numbers and the real numbers. Indeed mathematicians have a special interested in these two particular sets. An examination of the cardinality of the two sets, that is, the size of the real numbers versus the size of the natural numbers, produces a startling result. Based on Cantor's innovations and his explorations into the size of infinite sets, mathematicians refer to the "infinite" size of the set of all natural numbers. But, at the same time, the set of all real numbers is also infinite. Cantor's startling discovery was that these two infinities are different. Even more astounding, Cantor showed that the two infinities are not simply different, they have a different *size*, that is, there is no way to map a one-to-one relationship between each natural number and each real number. The cardinality of the natural numbers is of a different magnitude than that of the real numbers. A seeming paradox ensues, for how can infinity exist in two different magnitudes? Nevertheless Cantor demonstrated that the infinite size of the natural numbers will always be smaller than the infinite size of the real numbers.

Of the many repercussions produced by Cantor's theory, consider just one, the simple act of counting. Since the natural numbers are by definition the counting numbers, natural infinity is, by extension, countable, at least in principle. Yet if real infinity has a larger cardinality than natural infinity, then real infinity is "larger than countable," or more simply uncountable, innumerable. In other words, there is no way–no practical way but no theoretical way either–of counting all of the real numbers. With what tools would they be countable, now that the natural numbers are exhausted? The real numbers are innumerable, and thus the two number sets are numerically incompatible.

As a consequence of these discoveries, Cantor proposed in 1878 what he called the Continuum Hypothesis. The Continuum Hypothesis says, in essence, that the cardinality of the natural numbers is different from the cardinality of the real numbers, with no other set of numbers between them. Thus the natural numbers have one kind of infinity, natural infinity, while the real numbers have a larger kind of infinity, real infinity. And these two different kinds of infinity have two different "sizes." (Although the question of size starts to lose its meaning in this context, which is one reason why Cantor preferred the notion of cardinality to that of size.) With the former, natural infinity, it is possible to make a one-to-one correspondence with the counting integers, and thus the former is "countable." With the latter, real infinity, such a correspondence is not possible, and thus real infinity is quite literally uncountable.

The Continuum Hypothesis gets its name from "the continuum," that more poetic monicker for the real number line. Still, the

Continuum Hypothesis asserts an elemental *dis-continuum*, namely the insurmountable discontinuity between the natural and real numbers. According to the hypothesis no number exists *between* the cardinality of the natural numbers and the cardinality of the real numbers. According to Cantor, it is not possible to count continuously from the cardinality of the natural numbers "up" to the cardinality of the real numbers; a jump, is all, from one to the other. There exists a mathematical rift, as it were, a gap between numbers. (Cantor elegantly mapped this on to sets and their power sets, since the power set of the natural numbers will produce the real numbers, with an intermission between the two cardinalities.) More generally the hypothesis says–now following a looser interpretation–that there are two fundamental kinds of numbers, the natural kind and the real kind. These are not two different mathematics, as it were, but nevertheless two essentially different modes of number: natural and real with a fissure in between.

This impasse so captured Badiou's imagination that, as I have suggested, he structured *Being and Event* almost entirely around it, around what he called the errancy or the unmeasure of ontology. In Badiou's view Cantor unearthed "two regimes," mandating an "arbitrary decision" between them (278). In that momentous Meditation Twenty-Six, Badiou labeled this arbitrary decision a "wager" (*pari*) beyond the effectivity of known calculation. The English term wager does not entirely capture the meaning of Badiou's original *pari*. But the gist is that when calculation fails one is forced to gamble. One is obligated to make a choice, if not a leap of faith then a leap of faithfulness (*fidélité*). "A chasm opens" in the wake of Cantor, Badiou wrote, a chasm that requires "a conceptless choice" (280). If this sounds like existentialism, it should; Badiou is, in a sense, rewriting existentialism for a new age.

Yet, at this stage in *Being and Event*, Badiou has not yet turned to the work of Cohen in any real detail. Thus Badiou's "conceptless choice" is not a reference to the independence of the Continuum Hypothesis, at least not yet. There's something else, something within the hypothesis itself that provides Badiou with his initial fuel. The simple premise that the cardinality of the real numbers is qualitatively larger than the cardinality of the natural numbers–with no gradation between the two–this simple premise is enough to precipitate Badiou's "conceptless choice."

In the wake of the discoveries by these four mathematicians — Cantor-Gödel-Cohen-Easton — Badiou observed that "Being…is unfaithful to itself," and that, as a result, "quantity…lead[s] to pure subjectivity" (280). It is an astounding if not radical claim. Begin with quantity, with mathematical concepts; pursue their consequences far enough by following all the innovations of modern mathematics; and the result will be subjectivity. In other words, at some point Cantor's impasse will intrude, and one will encounter a point of decision, a point that is not quantifiable, a point that does not follow the succession of numbers. A yawning void will eventually open at the heart of mathematics, a void within mathematics, to be sure, but the consequence of mathematics nonetheless. And from out of this abyss, via the conceptless choice, the subject appears. The pursuit of quantity leads to subjectivity. In other words, *math makes subjects*. Such is the fundamental principle guiding all of Badiou's work as a philosopher. Its proper name shall be *mathification*.

taken from here

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